ALTERED QUEENS PROBLEM

Marla K. Ferguson

2017

MOREHEAD STATE UNIVERSITY

The Queen

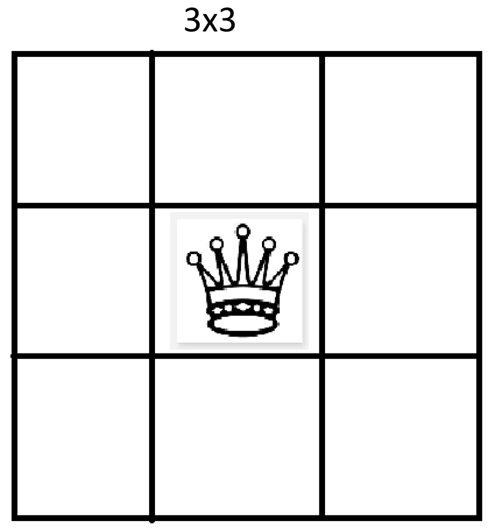
In a standard chess board there are 8 rows, 8 columns, and 6 different movable playing pieces. Each individual piece has its own special moves/routes, but the Queen seems to draw more attention than any other piece. What is so attractive about the queen?

In the game of chess, the queen is the most resourceful player because she can move more freely than any other piece. She can move forward, backward, and both diagonal directions as any times as she would like.

Queens Problem

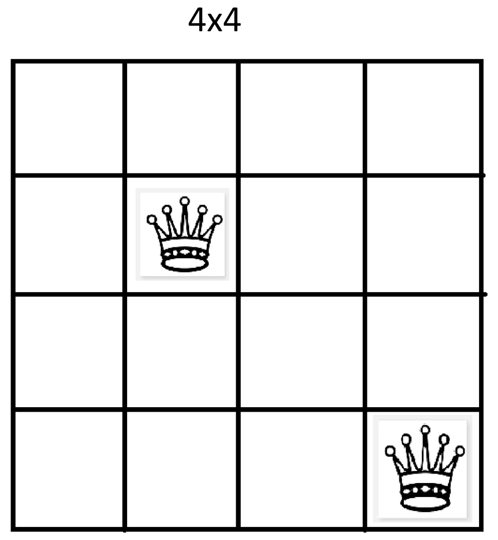
The current questions I have been exploring include: what is the minimum number of queens that can be placed on an NxN chessboard, so that every square on the board is attacked? Is there some sort of pattern that can be distinguished in placing the queens on a larger-sized board? Can I label each square and queen in such a way that I know exactly which queen is attacking a specific square on the board?

3x3



The minimum number of queens you can place on a 3x3 chessboard and still attack each square is one queen. If the queen is placed in the very center piece, then all 9 spaces can be attacked by moving in every direction possible.

4x4

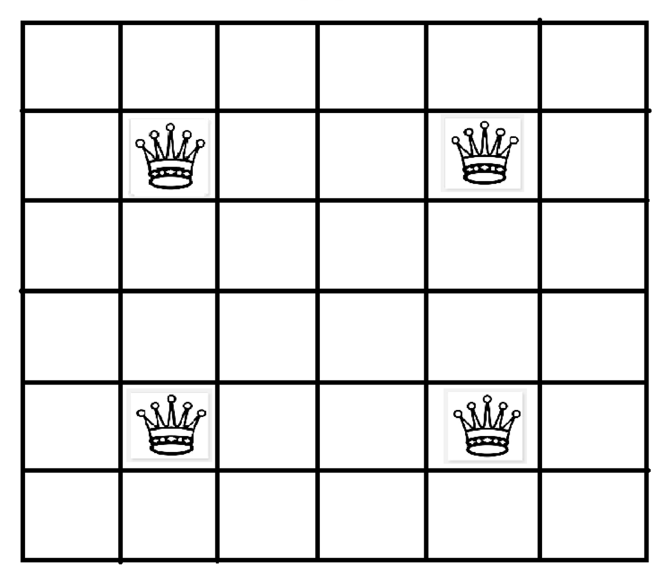


A minimum of 2 queens is required for a 4x4 board. When one queen is placed closest to the center, the maximum number of squares one queen can attack is 12 squares. Because there are 16 squares on a 4x4 board, there must be at least 2 queens.

General Formula

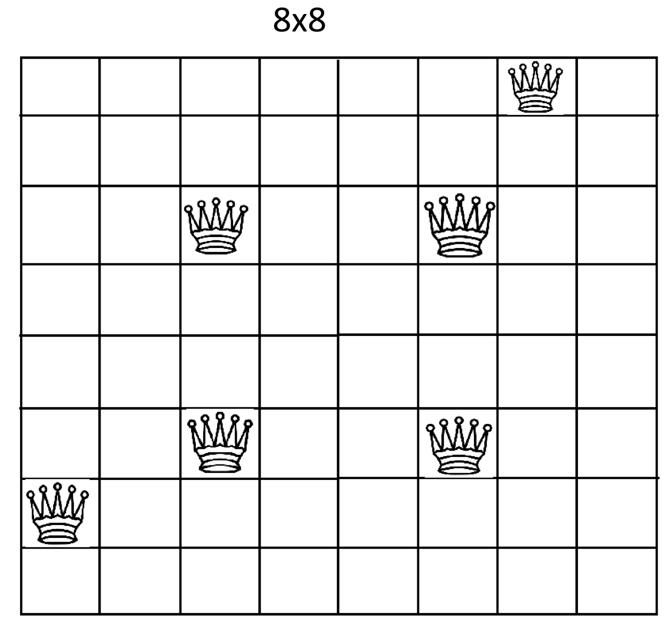
If N is an even number, then the maximum number of squares one queen can attack is equal to N+(N-1)+(N-1)+(N-2)= 4N-4, where the first N alone equals the row, the first (N-1) equals the column, the second (N-1) equals the diagonal, and (N-2) equals the other diagonal. Now, if N is an odd number, then the maximum number of squares one queen can attack is equal to N+(N-1)+(N-1)+(N-1)= 4N-3, following the same representation as the previous equation.

6x6



Using the previous formula, we see that 4N-4= 4(6)-4= 20 maximum spaces can be attacked by one queen. However, because of overlap in the squares, not all 36 can be attacked with two queens. Therefore, 4 queens can attack al 36.

8x8

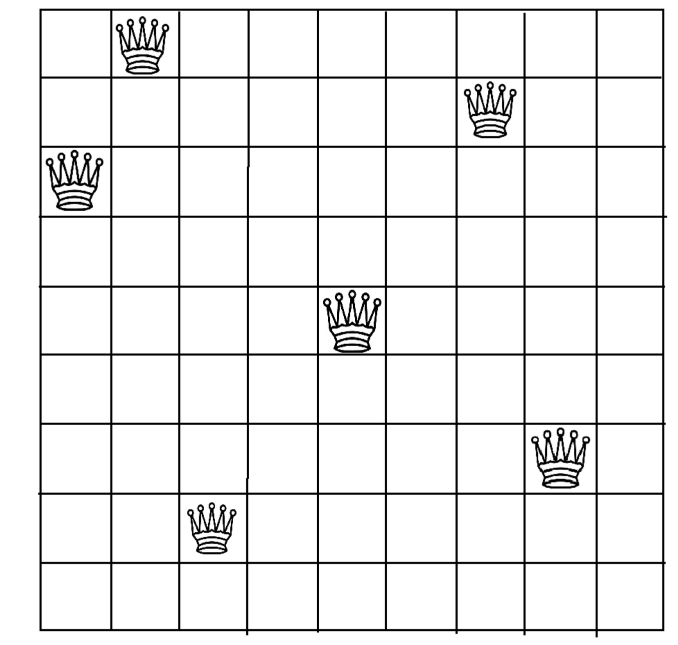


Using the general formula, one queen, if placed in the middle, can attack 4N-4 = 4(8)-4 = 28 spaces. 4 queens are arranged to form a square within the 8x8 board, then 2 more added along the outside. These last 2 can attack the leftover spaces that the inner 4 cannot.

Original Theory

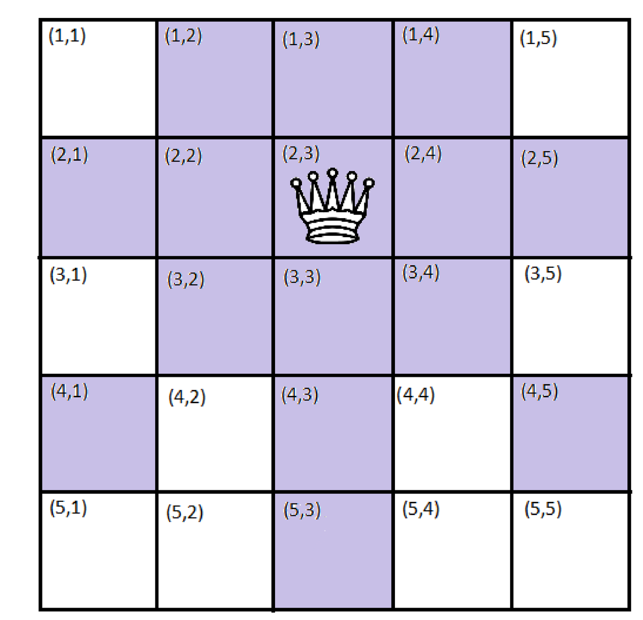
It appeared that the amount of queens needed to cover the board would always be (N-2) where N represents the NxN board. This theory seemed to hold until attempting the 9x9 case. The 9x9 board proved this theory to be false because (9-2) = 7 and the board was covered using 6 queens.

9x9



One queen in the center can attack 4N-3= 33 spaces. This board was the most challenging because previous strategies did not seem to apply, and it took fewer queens than expected to cover the entire board.

Index Coding

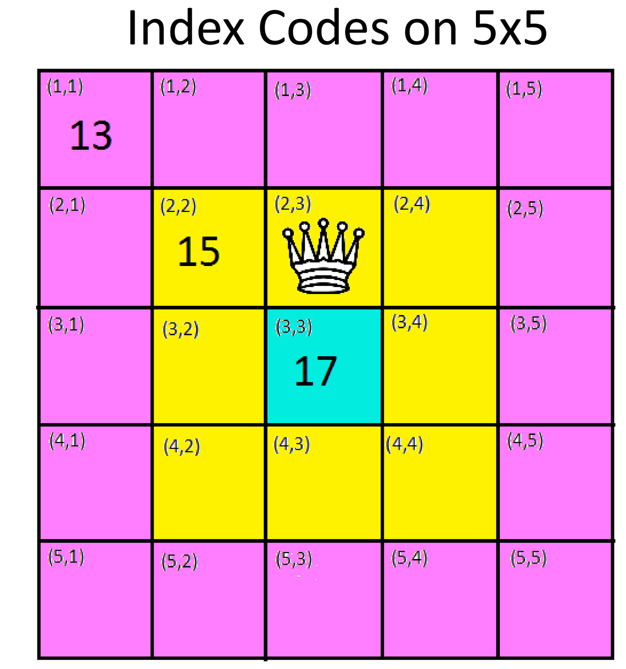


Each space is identified (row, column) as the ordered pair. By using this order system, which we will call index codes, we are able to pinpoint how many spaces one queen can cover, no matter where she is being placed.

The queen in (a,b) can attack:

- *(N-1) new spaces on any (a,i) row*
- *(N-1) new spaces on any (j,b) column*
- *spaces where i+j = a+b on the diagonal beginning at the top right to the bottom left*
- *spaces where j-i = b-a on the diagonal beginning at the top left to bottom right*

Index Codes on 5x5



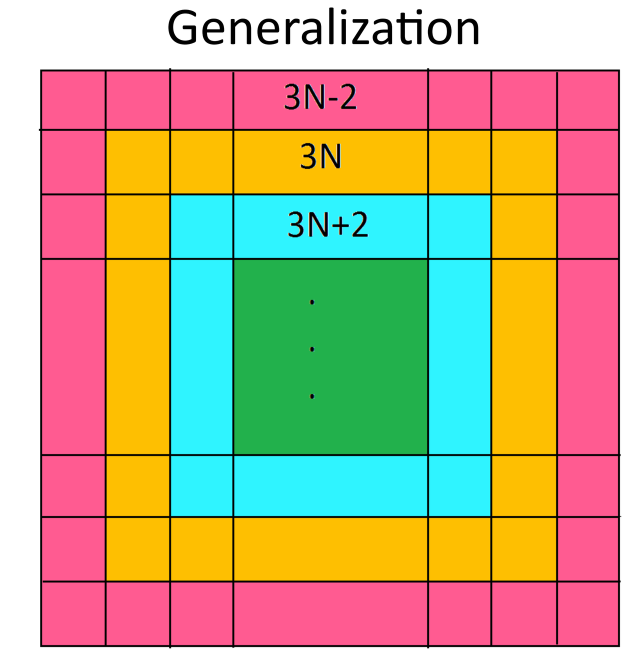
A queen placed in the following locations can attack:

Pink: attacks (N-1) = 4 diagonal spaces for a total of 3N-2 = 15-2 = 13 spaces

Yellow: attacks (N+1) diagonal spaces for a total of 3N = 15 spaces

Blue: attacks (N+3) 8 diagonal spaces (both directions inclusive, excluding her own space) for a total of (3N+2)= 17 spaces

Generalization



As the size of the board increases, queens placed in pink squares can attack 3N-2 squares. As we move into yellow, the queen can attack 3N squares and as we continue moving inward along the square frames, the number of squares attacked by a queen increases by 2.

Acknowledgements:

Advisors: Dr. Vivian Cyrus, Dr. Duane Skaggs